GET READY TO STUDY A LEVEL MATHEMATICS & FURTHER MATHEMATICS

Thank you for choosing to study A level Mathematics at Barnsley Sixth Form College.

Here is your checklist for getting started on A-Level Maths

- ☐ I have read through the course content and FAQs
- ☐ I have read through the Get Ready for A-Level Maths Booklet, the worked examples and understand how to do these topics
- ☐ I have completed the get ready for A-Level Maths work on pages 43-48 on separate paper and will bring this into college on my first week.
- ☐ **Further Maths Only** I have completed the further maths work that is at the end on separate paper and will bring this into college on my first week.

Mathematical techniques are important in solving problems in almost all areas of society today. Mathematics has been essential to the development of space travel, telecommunications, computing, power generation, and the understanding of environmental issues. The uses of mathematics are:

- to explain
- to predict
- to make decisions.

What follows next is key information about the course, guidance through the 27 key topics, the summer work for you to complete and the further maths work. Only complete the further maths work if you are interested in taking further maths.

If you are planning to study **A Level Maths** with us in September, please review this document and complete the required activities. For A Level Maths you will need to complete questions on paper showing all working out.

If you are also planning to study **A Level Further Maths** with us in September, please also complete the additional questions at the end of this section on separate paper, demonstrating all working out. Please bring the completed activities with you at induction.

FAQ

What specification will I study?

You will be studying the Pearson Edexcel A Level Mathematics specification.

Use the weblink to read about the specification and you can also view past papers to see what the assessment looks like.

A Level Mathematics Specification

How many lessons will I have a week?

You'll have two single lessons lasting 1 hour and one double lesson lasting between 1 hour and 30 minutes to 2 hours and 30 minutes per week depending on support session attendance and curriculum needs.

Who can I contact if I have a question about this subject?

Martin Hickman m.hickman@barnsley.ac.uk

Lesley Thorpe: l.thorpe@Barnsley.ac.uk

What subjects go well with Mathematics?

Maths is a challenging and demanding subject. We recommend you choose other STEM subjects such as biology, chemistry, physics, or computer science to complete your study programme. Other complementary subjects include economics and geography.

What grades should I have?

In addition to the general sixth form entry requirements, you must have a minimum of grade 7 or above in GCSE maths. If you get a grade 6 you will be given the opportunity to sit a maths skills assessment to demonstrate that A-Level maths will be right for you.

What extracurricular activities can I get involved with?

We will have a problem-solving group from September and students are welcome to set up their own group or society. In the past we have also had chess club and Further Maths for Fun. Every November we offer students the chance to take part in the Senior Mathematical Challenge UK and National Problem Solving competitions such as Ritangle and MathsBombe.

WHAT WILL I STUDY?

Over the two years you will study Pure Mathematics (which makes up two-thirds of the course), Statistics (one-sixth) and Mechanics (one-sixth). The course is examined by three 2-hour exams at the end of the two-year course. Papers 1 and 2 assess Pure Maths and Paper 3 assesses the Statistics and Mechanics element.

Pure Maths

Pure Mathematics provides the formulae, techniques, and basic knowledge required in applying Mathematics in the real world. Pure Mathematics starts by reviewing, consolidating, and developing knowledge achieved at GCSE level, and then progresses to new topic areas:

- Algebraic Techniques
- Functions
- Sequences and Series
- Co-ordinate Geometry
- Trigonometry
- Differential Calculus
- Integral Calculus
- Numerical Methods

Statistics

Statistics deals with the collection of data and its analysis, presentation, and interpretation together with calculating probabilities and using probability distribution models. Statistical techniques and the use of mathematical models are now used throughout modern industrial societies for analysis of data, to explain observations, and to help make rational decisions. Statistics also starts by consolidating knowledge achieved at GCSE level and then progresses to new topic areas:

- Representing data
- Summary statistics
- Correlation and regression
- Sampling techniques
- Probability
- Random Variables
- Discrete distributions
- Continuous distributions

Mechanics

Mechanics applies Pure Mathematics to problems encountered in the real world by modelling situations which involve the behaviour of bodies or particles under the action of a system of forces. For example, the solution of the equations of motion for ballistics has helped design rockets of awesome power which can pinpoint targets. Mechanics develops and uses Pure Mathematics techniques in simplified real-world problems.

- Vectors
- Kinematics
- Projectiles
- Dynamics of a particle
- Statistics of a particle
- Moments

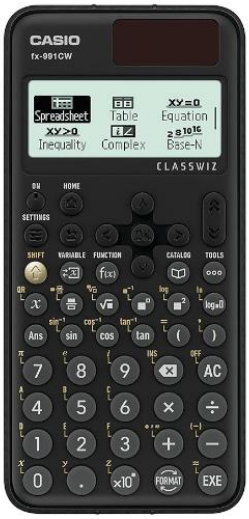
WHAT WILL I NEED?

To study A level Maths, you need to purchase a **pencil case** and fill it with the following essential equipment:

- **Black pen (more than one)**
- **Coloured pens (green, blue, red, etc.)**
- **Pencil, rubber, and sharpener**
- **Ruler and highlighters**

You also need to purchase a **lever arch file** and **ring binder** for each of your subjects. You will be given worksheets, course notes, and booklets to complete over the two-years and these must be stored in a well-organised file with **dividers**. Unlike school where you may have been given exercise books to write in, for A level study, you will need to bring paper to write on. For maths we recommend:

- **A4 squared paper (lined paper is acceptable if not)**

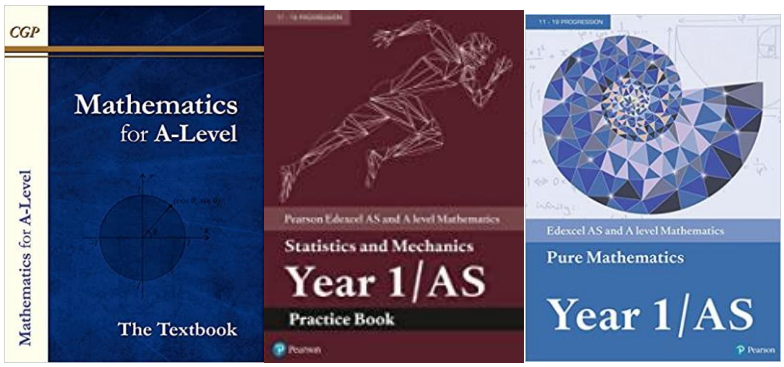


It is now a course requirement that all students use a specific calculator for A level Maths. You will need to purchase the **CASIO fx-991CW** (please be careful it is this exact code; the shift button must be gold) either before the start of the course or during the first few weeks of the term*. This version has certain functions and settings needed for A level that the calculator you used at GCSE may not have. These can be purchased for around £30 from supermarkets (Tesco, ASDA, etc.) and online stores (Amazon). **When buying online please check that you are purchasing the UK version of the calculator.**

Students studying A level maths also find it useful to have:

- **Blank flashcards (or coloured cards)**
- **A mini-whiteboard and pens**

All students will be given a login to use online textbooks for the course that can accessed from anywhere, at any time. Most of your homework will be accessed from the online textbooks.
Textbooks and study guides are also available to buy*.



*Financial support is available for those learners that need help with study costs.

FIND OUT MORE

Use the suggested links in this section to find out more about the wonderful world of maths.

Careers information

Our students have gone on to study a wide range of courses at university and enter a number of exciting careers and job roles. Maths is a facilitating subject, meaning that it can open a lot of doors for you. Employers love maths graduates!

- What can I do with a maths degree?
- Maths careers
- Business and finance
- Exploring STEM careers
- Careers in mathematics and statistics

YouTube channels

There are many interesting and useful videos that will help to broaden your understanding and enjoyment of maths. We recommend subscribing to the following channels:

- 3Blue1Brown
- Numberphile
- Eddie Woo
- Stand-up Maths

Useful maths revision/preparation links

These links are designed to support your understanding of GCSE maths in preparation for studying this subject at an advanced level.

- Physics & Maths Tutor
- TLMaths
- DrFrostMaths
- MathsMadeEasy
- Maths Genie

Maths Films

There are some wonderful mathematical films (many based on true historical people and events) such as:

- *Hidden Figures*
- *The Imitation Game*
- *A Beautiful Mind*
- *The theory of Everything*
- *The Man Who Knew Infinity*
- *X+Y*
- *Good Will Hunting.*

Further reading

There are loads of great mathematical books you can read. Here are a few of our favourites:

- *50 Mathematical Ideas You Really Need to Know* (Tony Crilly)
- *Alex's Adventures in Numberland* (Alex Bellos)
- *Cabinet of Mathematical Curiosities* (Ian Stewart)
- *The Calculus Wars* (Jason Socrates Bardi)
- *The Code Book* (Simon Singh)
- *How Many Socks Make a Pair?: Surprisingly Interesting Maths* (Rob Eastway)
- *Hello World: How to be Human in the Age of the Machine* (Hannah Fry)
- *Humble Pi: A Comedy of Maths Errors* (Matt Parker)
- *The Life-Changing Magic of Numbers* (Bobby Seagull)
- *The Num8er My5teries* (Marcus du Sautoy)
- *Fermat's Last Theorem* (Simon Singh)
- *The Simpsons and their Mathematical Secrets* (Simon Singh)

GET READY FOR A-LEVEL MATHS WORK BOOKLET

A-Level Maths relies upon a lot of the algebra that you learnt at GCSE. In order to expand on these concepts and learn new ones, it is important that you have these skills in place before you start as we won't have the time to teach GCSE content with all the new content coming up.

In this booklet there are 27 key topics that you need to be prepared for before starting A-Level. Don't worry if you've forgotten how to do some of these, there are worked examples guiding you through the topics. If you are not sure about any of these topics, it's important to revisit these throughout the summer so you are prepared for A-Level Maths. At the end of this booklet are questions for you to attempt based on these 27 key topics. You need to complete this work on separate paper ready to hand in on your first week at college.

In the second week there will be a test on these skills to see if A-Level maths is the right course for you.

What you need to do:

- ☐ Read through the booklet and check you are confident with all these skills in this booklet
- ☐ Complete the work on pages 43-48

The boxes for the worked examples have been left intentionally blank so that you can attempt these yourself first. If you would like to check your answers, you can do so here:

https://drive.google.com/drive/folders/1VNY_k7H3wiZp840IJl7d7jNNpkbnc17S?usp=sharing

Found a mistake? Please e-mail m.hickman@barnsley.ac.uk

Note that this only for the worked examples. The solutions for the work you need to hand in are not given.

1. Expanding Brackets

To find the product of two expressions, you multiply each term in one expression by each term in the other expression.

Worked Examples 1

Expand and simplify the following

a) $3(x-2)+5(3-x)$ b) $(2x+1)(x-5)$ c) $x(3x+4)-7(2-3x)$

d) $(x+2y)(x^2-3)$ e) $(x-3y)^2$ f) $(x+y)(3x-4y+5)$

2. Expanding Brackets with more than two expressions

If there are more than two expressions, you will need to multiply two together first and then multiply this by the third. With multiplication, this can be done in any order.

Worked Examples 2

Expand and simplify the following:

a) $x(3x + 4)(x - 4)$
b) $x(6x - 2y)(x - y + 5)$
c) $(2x + 1)(x - 5)(x + 4)$

3. Solving Linear Equations

From GCSE we will assume that you can solve simple linear equations. This section will review some the harder linear equations that involve fractions and multiple steps. As always with equations, remember that in order to keep the equation balanced, we must perform the same operation to both sides.

Worked Examples 3

Solve the following equations:

a) $\frac{5-x}{2} = 2x - 7$ b) $5x - 6 = 3(x - 1)$ c) $\frac{3x-2}{4} - \frac{2x+5}{3} = \frac{1-x}{6}$

4. Changing the subject

Worked Example 4.1

Make *v* the subject of the formula $w = \frac{15(t-2v)}{v}$

Worked Example 4.2

Make *m* the subject of the formula $f = \frac{3m+4}{m-1}$

Worked Example 4.3

Make t the subject of $p = \sqrt{a + \frac{t}{2}}$

5. Factorising

We can write expressions as a product of their factors. Factorising is the opposite of expanding.

Worked Examples 5

Factorise these expressions **completely.**

a) $12x - 6$

b) $x^3 - x^2$

c) $14x^2 - 20x$

d) $9x^2y + 15xy^2$

e) $3x^2 - 9xy$

6. Factorising Quadratics and Difference of Two Squares

A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and a is non-zero. You will often need to factorise these to solve equations.

A common way to solve a quadratic expression is to do the following:

- Find two factors of ac that add up to b
- Rewrite the b term as a sum of these two factors
- Factorise each pair of terms
- Take out the common factors.

However, there are other ways and if you have a valid method that you learnt at GCSE, you can stick with that. It's always worth expanding back out the brackets to check your factorisation was correct!

Another common expression you will come across are expressions in the form $x^2 - y^2$. This is known as the **difference of two squares** and when you factorise this it will become:

$$x^2 - y^2 = (x + y)(x - y)$$

Worked Examples 6

Factorise:

a) $x^2 - 5x + 6$
b) $x^2 + 3x - 10$
c) $2x^2 - 5x + 3$
d) $x^2 - 49$
e) $4x^2 - 9y^2$
f) $2x^3 + 12x^2 + 16x$

7. Laws of Indices

Suppose we want to simplify $3x^4 \times 2x^5$, we could write it all out:

$$\begin{aligned} 3x^4 \times 2x^5 &= 3 \times x \times x \times x \times x \times 2 \times x \times x \times x \times x \times x \\ &= 3 \times 2 \times x \times x \times x \times x \times x \times x \times x \times x \times x \\ &= 6x^9 \end{aligned}$$

In a similar fashion, suppose we want to simplify $8x^7 \div 2x^2$:

$$\begin{aligned} 8x^7 \div 2x^2 &= \frac{8x^7}{2x^2} \\ &= \frac{8 \times x \times x \times x \times x \times x \times x \times x}{2 \times x \times x} \\ &= 4x^5 \end{aligned}$$

Now consider simplifying $(5x^2)^3$:

$$\begin{aligned} (5x^2)^3 &= (5x^2) \times (5x^2) \times (5x^2) \\ &= 5 \times 5 \times 5 \times x \times x \times x \times x \times x \times x \\ &= 125x^6 \end{aligned}$$

However, we can save time with the rules on the next page.

General rules:

$\boldsymbol{a^n \times a^m = a^{n+m}}$	$\boldsymbol{(a^n)^m = a^{n\times m}}$
$\boldsymbol{a^n \div a^m = a^{n-m}}$	$\boldsymbol{(ab)^n = a^n b^n}$

Worked Examples 7

Simplify the following:

a) $5x^7 \times 6x^2$ b) $\frac{2y^7}{6y^4}$ c) $(a^3)^4 \times 2a^5$ d) $\frac{(2x)^5}{4x}$

8. Indices – Fractional and Negative Powers

We also need to deal with negative and fractional indices. There is also an important rule that anything to the power of 0 is always 1.

$a^0 = 1$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
$a^{-n} = \frac{1}{a^n}$	$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$

Worked Examples 8

Simplify the following:

a) $\frac{x^4}{x^{-2}}$ b) $x^{\frac{1}{2}} \times x^{\frac{1}{3}}$ c) $\left(x^{\frac{1}{5}}\right)^3$ d) $\sqrt{64x^6}$ e) $\sqrt[3]{125x^6}$ f) $\frac{x^2}{\sqrt{x}}$

9. Writing in the form ax^b

We often want to write terms in the form $\boldsymbol{ax^b}$ where a and b are constants (numbers that don't change). This is super important for an A-Level topic later on!

To do this, we should deal with the numbers first and then deal with the indices. We need to make sure there are no variables on the bottom of any fractions, so the $\frac{1}{x^n} = x^{-n}$ rule is really useful here!

Worked Examples 9

Write the following in the form ax^b where a and b are constants.

a) $\frac{1}{x^3}$ b) $\frac{1}{2x}$ c) $\frac{5}{\sqrt{x}}$ d) $\frac{x}{4x^3}$ e) $\frac{3\sqrt{x}}{5x^2}$

10.Indices and Fractions

It's quite common that you will see fractions where there are multiple terms on the numerator and a single term on the denominator. For these problems you want to make sure every term on the numerator is divided by the term on the denominator. It is often helpful to separate out the fraction.

Worked Examples 10

Simplify the following leaving all terms in the form ax^b where a and b are constants.

a) $\frac{5x+3x^2}{15x^3}$ b) $\frac{3-5x^3}{\sqrt{x}}$ c) $\frac{(2x+3)^2}{\sqrt{x}}$

11. Simplifying Surds

A **surd** is the root of a whole number that has an irrational value (e.g. $\sqrt{2}$, $\sqrt{5}$, $\sqrt{7}-2$, $\sqrt[3]{5}$). It is often necessary to manipulate expressions with surds.

An **irrational number** is a number that cannot be written as a fraction with an integer numerator and denominator. (Therefore, a rational number is a number that can be written as a fraction)

Worked Examples 11

Simplify each of the following expressions

a) $\sqrt{12}$

b) $\frac{\sqrt{40}}{2}$

c) $\sqrt{75}+3\sqrt{80}-2\sqrt{12}-2\sqrt{20}+5\sqrt{45}$

d) $(1+\sqrt{2})(3-\sqrt{3})$

e) $(\sqrt{5}+\sqrt{2})(\sqrt{3}+\sqrt{5})$

12.Rationalising the denominator

Sometimes we have a fraction where the denominator is a surd. For example.

$$\frac{1}{\sqrt{2}}, \qquad \frac{1}{1+\sqrt{3}}, \qquad \frac{2+\sqrt{7}}{3-\sqrt{7}}.$$

Try to input either of these three into your calculator – what do you notice?
Your calculator is automatically rearranging the expression to avoid having a surd in the denominator – this is called **rationalising the denominator.**

In your exams you will be expected to **demonstrate** that you can use surds and recognise when and how to rationalise the denominator. So you must show your working rather than just rely on your calculator.

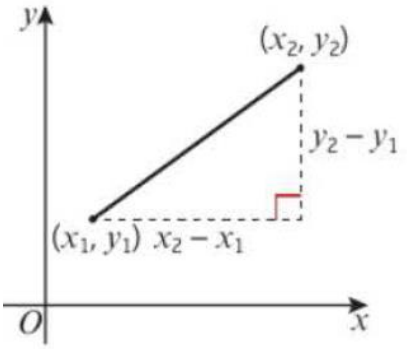
Worked Examples 12

Rationalise and simplify the following fractions,

$$\frac{2}{\sqrt{3}} \qquad \frac{8\sqrt{3}}{\sqrt{5}} \qquad \frac{1}{3+\sqrt{2}} \qquad \frac{4}{\sqrt{7}-\sqrt{5}} \qquad \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \qquad \frac{3}{\left(\sqrt{3}-1\right)^2}$$

13. Finding the gradient

We can find the **gradient** of any straight line joining two points by dividing the vertical distance between the points by the horizontal distance between them.



$$\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Worked Example 13.1

Given the coordinates $A(-2, 1)$ and $B(4, 5)$, find the gradient of line AB.

Worked Example 13.2

The line joining coordinates $C(2, -5)$ and $D(4, a)$ has gradient -1. Find a.

Worked Example 13.3 The line joining coordinates $E(-5, 8)$ and $F(b, -3)$ has gradient $\frac{2}{3}$. Find b.

14. Equation of a Line

The equation of a straight line can be written in two different forms:

- $\boldsymbol{y = mx + c}$, where m is the gradient and c is the y-intercept.
- $\boldsymbol{ax + by + c = 0}$, where a, b and c are integers.

Worked Example 14.1 Write down the gradient and y-intercept of these lines

$y = -6x + 4$	$6x - 3y + 2 = 0$

Worked Example 14.2
Write these lines in the form $ax + by + c = 0$, where a, b and c are integers.

$y = -2x + 1$	$y = -\frac{1}{3}x + 5$	$y = \frac{1}{2}x - \frac{7}{4}$

15. Roots of Lines

We can find where a line meets the x-axis (i.e. find a root) by letting $y = 0$.

Worked Examples 15 Find the coordinates of the point where these lines meet the x-axis.

$y = 3x + 5$	$2x - 3y + 6 = 0$

16. Finding the equation of a line

We can **find an equation of a straight line** if we know either one point on the line and the gradient, or two different points on the line. We can use this rule to help us:

$$\boldsymbol{y - y_1 = m(x - x_1)}$$

You can still use $y = mx + c$ if you prefer.

Worked Example 16.1

Find the equation of the line with gradient 3 that passes through the point $(-1, 2)$.

Worked Example 16.2

Find the equation of the straight line passing through the points $(4, 1)$ and $(2, -3)$.

Worked Example 16.3

Find, in the form $ax + by + c = 0$, the equation of the line passing through the points $\left(1, -\frac{1}{2}\right)$ and $(3, -2)$, where a, b and c are all integers.

17. Sketching Lines

To sketch a line we need to know where it crosses the coordinate axes. You need to label these points on your sketch. When you do a sketch you do not need to draw a scale and have your sketch be to scale. Only show the key information.

Here are some useful facts for helping you to not only sketch lines but other sketches you will do throughout A-Level Maths

- A graph **crosses the x-axis** when **y = 0**
- A graph **crosses the y-axis** when **x=0**

Worked Examples 17 Sketch the following lines.

$2x - 4y + 12 = 0$
$2x + 7y - 1 = 0$

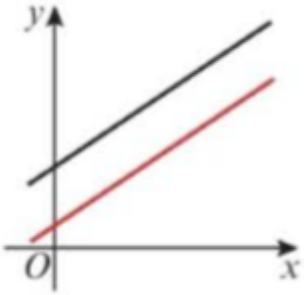
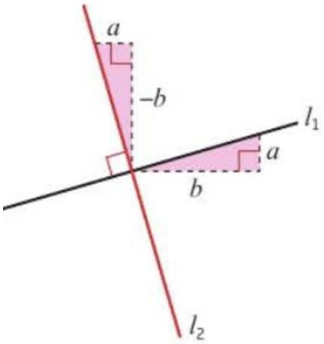
18. Parallel and perpendicular lines

Two lines are **parallel** if they have the same gradient: $\boldsymbol{m_1 = m_2}$

Two lines are **perpendicular** if the following is true: $\boldsymbol{m_1 \times m_2 = -1}$.

So if a line has gradient $\boldsymbol{m}$, a line perpendicular to it will have the gradient $-\frac{\boldsymbol{1}}{\boldsymbol{m}}$

This is known as the negative reciprocal.

Parallel	Perpendicular
	

Here are some examples of parallel and perpendicular gradients.

Gradient of line	Parallel gradient	Perpendicular gradient
4	4	$-\frac{1}{4}$
$-\frac{4}{3}$	$-\frac{4}{3}$	$\frac{3}{4}$
$\frac{1}{5}$	$\frac{1}{5}$	-5

Worked Example 18.1

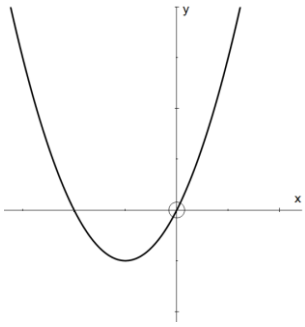
Find the equation of the line that is parallel to $y = 2x - 3$ and passes through $(3, 7)$.

Worked Example 18.2

A line perpendicular to $y = 2x - 3$ that passes through $(5, -1)$. Giving your answer in the form $ax + by + c = 0$, where a, b and c are integers, find the equation of this line.

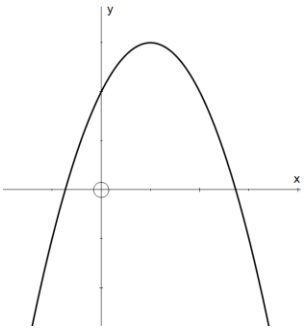
19. Solving Quadratic Equations via. Factorising

A **quadratic equation** is any equation where the highest power of x is 2. The general form of a quadratic equation is $y = ax^2 + bx + c$, where a, b and c are constants, with $a \neq 0$. We call a, b and c the **coefficients**. A quadratic curve is called a parabola. The graph of any quadratic equation has one turning point (the vertex) and cuts the x-axis at most twice.



$a > 0$

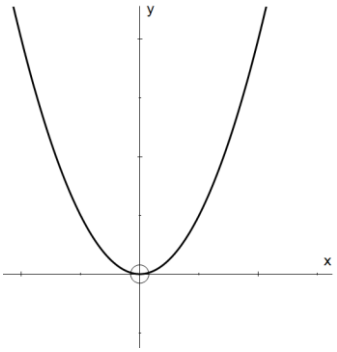
Positive Quadratic



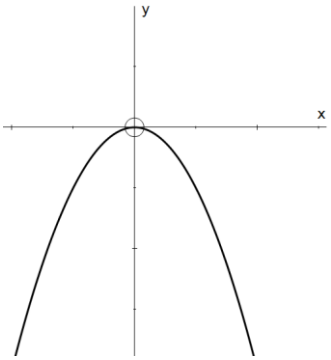
$a < 0$

Negative Quadratic

Key quadratic graphs you need to know:



$y = x^2$



$y = -x^2$

The points where a graph crosses the x-axis are known as the **roots** of the graph. There are three techniques that we will use for solving quadratic equations:

- **Factorisation**
- **Using the quadratic formula**
- **Completing the square**

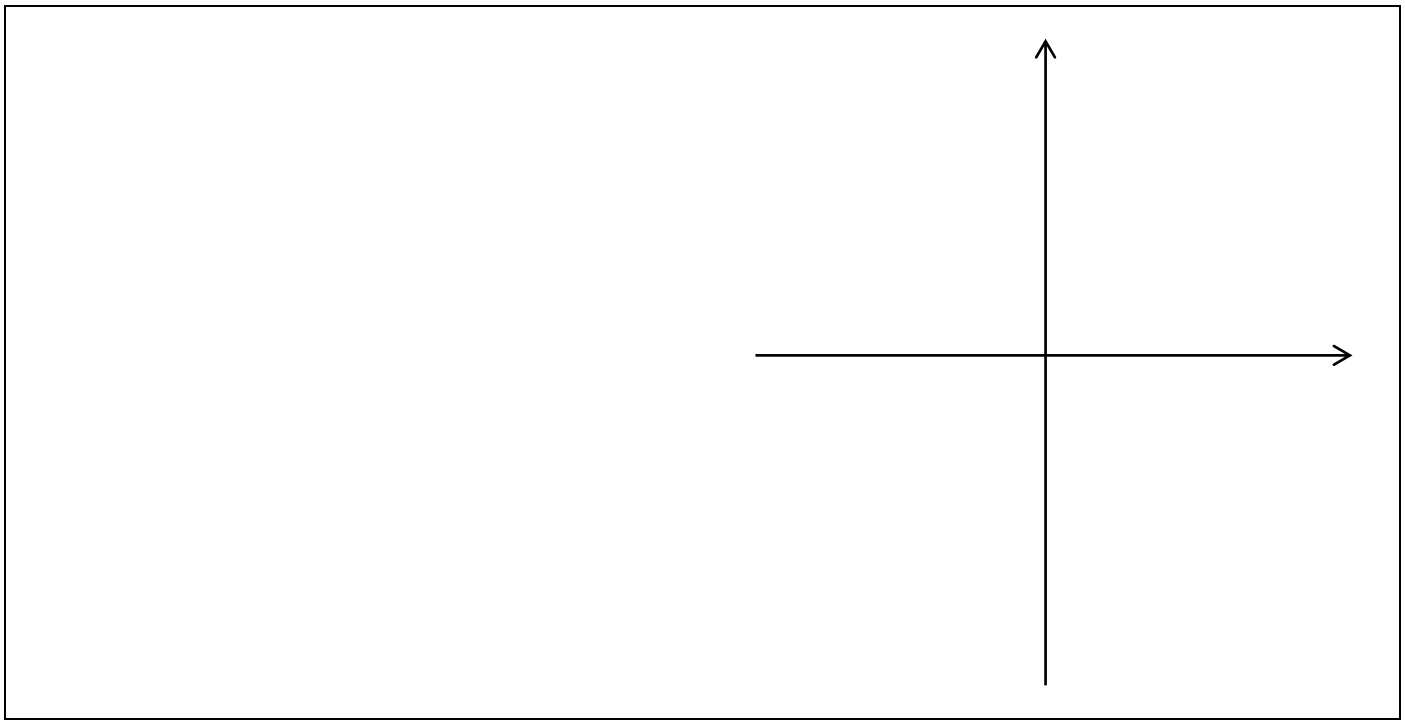
Worked Examples 19 Using factorisation, solve the following equations

$x^2 + 7x + 10 = 0$	$x^2 + 3x - 18 = 0$
$x^2 + 2x = 0$	$x^2 - 64 = 0$
$16 + x^2 = 10x$	$2 - x - x^2 = 0$
$5x^2 - 8x - 4 = 0$	$6x^2 + 5x - 4 = 0$

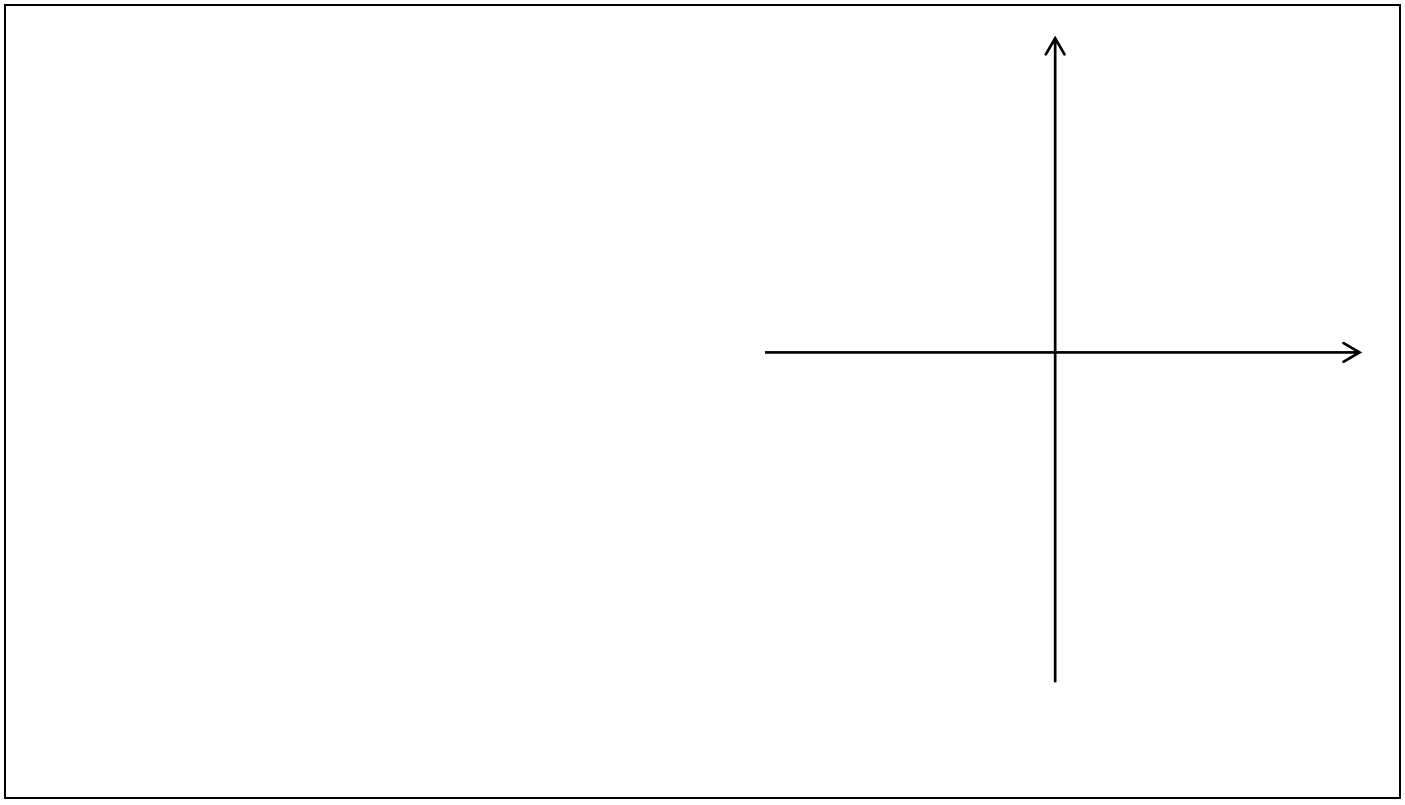
20. Sketching Quadratics

Worked Examples 20

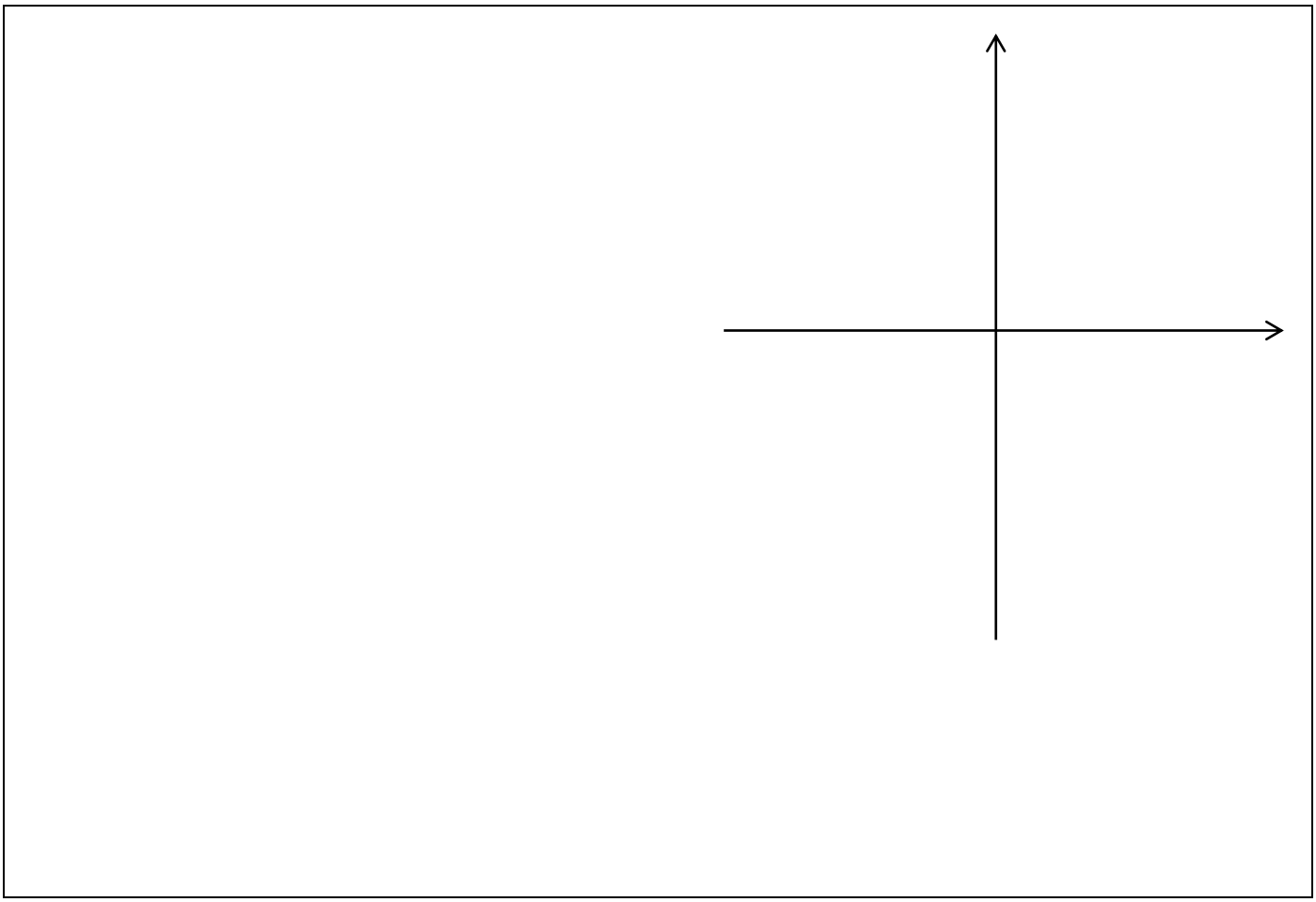
Sketch the graph of $y = x^2 - 9x$



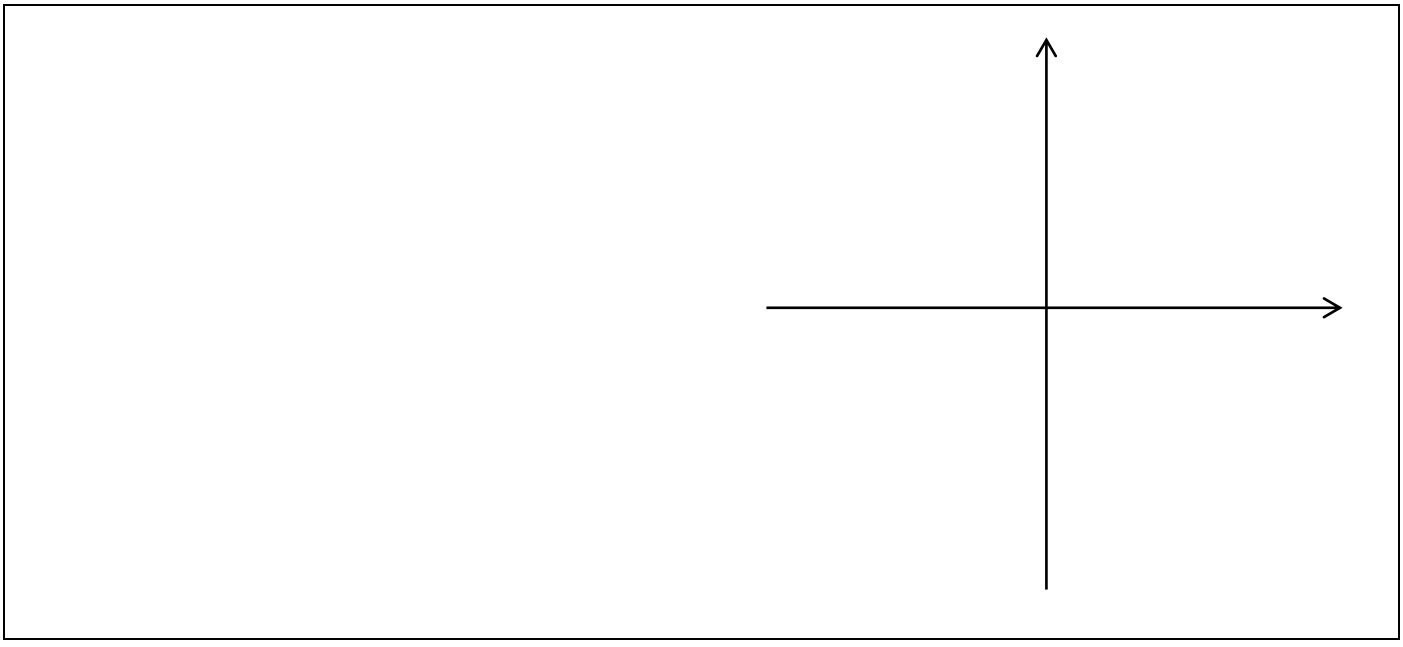
Sketch the graph of $y = -x^2 - 4x + 12$.



Sketch the graph of $y = 8x^2 - 10x - 3$.



Sketch the graph of $y = 4x^2 - 49$.



21. The Quadratic Formula

Sometimes a quadratic equation cannot be factorised and so to find the roots of the equation, we can use the **quadratic formula**:

For $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In an exam, always write down at least the first line of your working out so that they can see what method you are using. You can sometimes use your calculator to solve but this is not always the case!

Worked Example 21.1 Solve the equation $5x^2 + 6x + 1 = 0$ using the quadratic formula.

Worked Example 21.2 Solve the equation $10x^2 - x - 2 = 0$.

Worked Example 21.3 Solve $4x^2 - 8x = 2$, leaving your answers in simplified surd form.

22. Completing the square

Writing a quadratic expression $ax^2 + bx + c$ in the form $\boldsymbol{p(x+q)^2 + r}$ is called **completing the square** (or CTS for short). We begin with the following rule:

$$\boldsymbol{x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2}$$

Worked Examples 22.1 Complete the square for the following expressions

$x^2 + 8x$	$x^2 - 5x$	$2x^2 + 20x$

Worked Examples 22.2 Complete the square for the following expressions

$x^2 + 8x + 10$	$x^2 - 6x + 1$
$2x^2 - 8x + 12$	$-x^2 - 8x + 1$

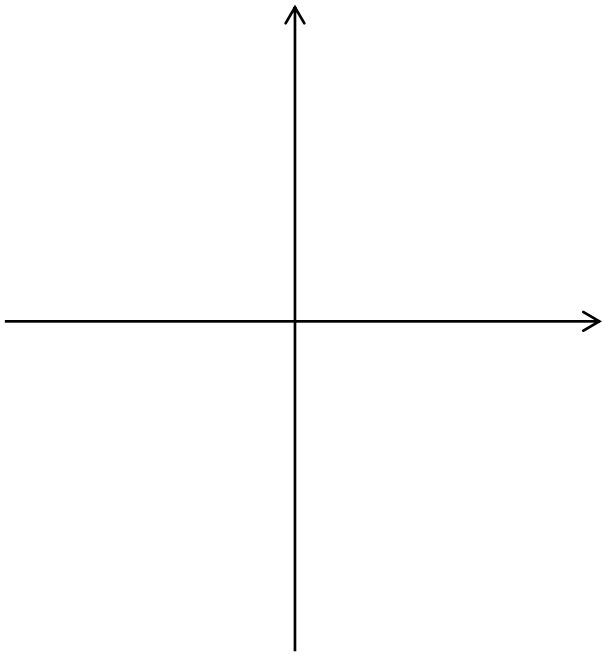
23. Finding turning points

Completing the square also helps us find the **turning point** of a quadratic. For a quadratic written in the form $\boldsymbol{p(x+q)^2+r}$ the coordinates of the vertex (or turning point) are:

$$maximum\ or\ minimum\ point\text{: } (-\boldsymbol{q}, \boldsymbol{r})$$

Worked Example 23

Find the turning point of the graph $y = x^2 + 4x - 5$ and state the line of symmetry.



24. Solving quadratics by completing the square

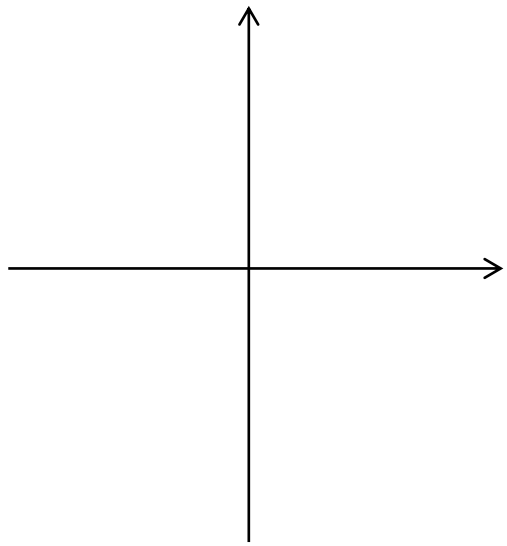
Completing the square can also be used to solve quadratics.

Worked Examples 24

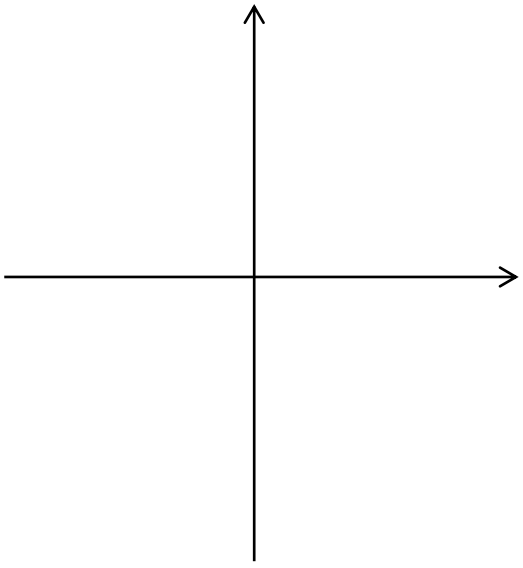
For the following equations:

(a) Solve by completing the square, leaving your answers in surd form.
(b) Hence sketch the graphs, labelling points of intersection with the axes and the vertex.

$x^2 + 8x + 10 = 0$



$-2x^2 + 8x - 7 = 0$



25. Solving linear simultaneous equations

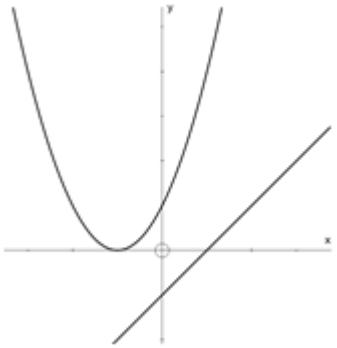
When we solve simultaneous equations, the x and y values we obtain are the **coordinates** of where the graphs of the two equations intersect. Solving two linear equations simultaneously will give one x value and one y value – just one point of intersection.

Worked Examples 25 Solve the following pairs of equations

$2x + 3y = 2$ $4x - y = -3$	$y = 2x + 4$ $x = y + 1$

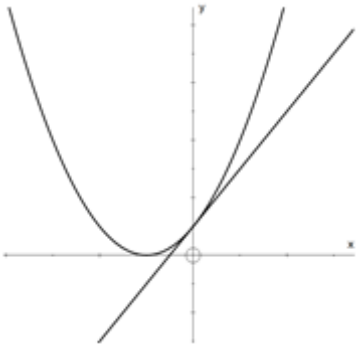
26. Simultaneous equations with one linear equation and one non-linear equation.

We can also solve simultaneous equations where one is linear and one is non-linear. It is often common to solve equations where one is linear and the other is a quadratic. By solving such equations, we should find either zero, one or two points of intersection:



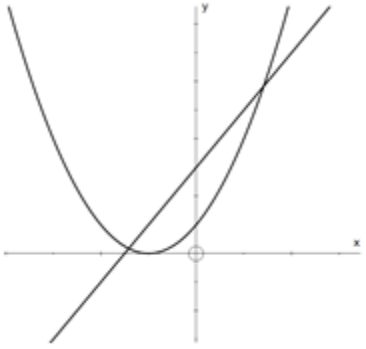
No point of intersection

The line misses the curve altogether



One point of intersection

The line is a **tangent** to the curve



Two points of intersection

The line cuts the curve in **two distinct** points

Simultaneous equations with one linear and one quadratic equation are usually solved by substitution using the following steps:

- Rearrange the linear equation to give one variable in terms of the other
- Substitute this into the non-linear equation

Worked Example 26.1 Solve $y = x^2$ $x - y + 2 = 0$

Worked Example 26.2 Solve $2x + y = 1$ $x^2 + y^2 = 1$

Worked Example 26.3 Solve $x + y = 9$ $x^2 - 3xy + 2y^2 = 0$

27. Solving linear inequalities

You can solve linear inequalities using similar methods to those for solving linear equations. When solving an inequality, the solution will be a range rather than one distinct solution. Recall:

$x > y$ means x is **greater than** y
$x \geq y$ means x is **greater than or equal to** y
$x < y$ means x is **less than** y
$x \leq y$ means x is **less than or equal to** y

For inequalities we have to be careful when we multiply through by a **negative**. Consider:

$$6 > 4 \quad \text{but} \quad -6 < -4$$

As you can see, when we multiply through by a negative, the inequality sign must change round. In general:

If $-x > a$, then $x < -a$

Worked Examples 27 Find the set of values for which

$2x + 3 < 8$	$6 - 2(x + 1) \leq 3(1 - 2x)$

INDUCTION TASK

Choosing which combination of A Levels to study can be a challenge for many students. Completing these activities will give you a greater insight into what is expected at the start of A level Maths and to help ensure you have made the right choice.

Interested in taking A level Further Maths? There is a separate set of questions at the end of this document. These need to be completed in addition to the A level Maths questions.

QUESTIONS TO SUBMIT (FOR A LEVEL MATHS)

The work we would like you to complete over the summer break is to continue to build on the algebraic problem-solving skills at grades 8 and 9 from GCSE. It is assumed that students beginning the course will have strong algebra and problem-solving skills. Some important early topics are algebra, equations, and graphs. **There will be a formal assessment in the second week of lessons**, so please use this task as an opportunity to review and practice these fundamental building blocks.

IMPORTANT: Formulae you were given for GCSE that you will be expected to know from memory at A Level:

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium $= \frac{1}{2}(a+b)h$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

Circumference of a circle $= 2\pi r = \pi d$

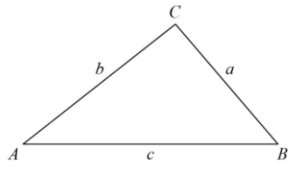
Area of a circle $= \pi r^2$

Quadratic formula

The solution of $ax^2 + bx + c = 0$ where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry



In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$

Please write your full solutions on A4 paper to hand in at induction.

Get Ready for A-Level Mathematics

You will need to bring this work in for your first lesson in college. Complete this work on seperate A4 paper

1. Expanding Brackets

Expand and simplify the following:

a) $4(2x-3)+2(5-x)$

b) $(3x-2)(x+4)$

c) $2x(5x-1)-3(4-2x)$

d) $(2x-y)(x^2+4)$

e) $(2x+5y)^2$

f) $(x-2y)(2x+3y-1)$

2. Expanding Brackets with more than two expressions

Expand and simplify the following:

a) $x(2x-1)(x+3)$

b) $2x(3x+y)(x-2y+1)$

c) $(x+2)(x-3)(2x+1)$

d) $(3x-1)(x+4)^2$

3. Solving Linear Equations

Solve the following equations:

a) $\dfrac{3-2x}{5}=x+4$

b) $4(2x-3)=2(x+5)$

c) $\dfrac{2x+1}{3}-\dfrac{x-4}{2}=\dfrac{5}{6}$

d) $\dfrac{4x-3}{2}+\dfrac{x+1}{4}=3$

4. Changing the subject

Rearrange the following formulas to make the indicated letter the subject:

a) Make x: $y=\dfrac{2x+3}{x-5}$

b) Make k: $T=\dfrac{4(k-3m)}{k}$

c) Make x: $y=\sqrt{\dfrac{x+1}{3}}$

d) Make v: $E=mgh+\dfrac{1}{2}mv^2$

5. Factorising

Factorise these expressions completely:

a) $15x+20$

b) $2x^4-6x^3$

c) $18x^2y-24xy^2$

d) $5x^3-15x^2y$

e) $4x^2y^2+12xy^3-8x^3y$

6. Factorising Quadratics and Difference of Two Squares

Factorise the following completely:

a) $x^2 - 8x + 15$

b) $x^2 + 2x - 24$

c) $3x^2 + 11x - 4$

d) $x^2 - 81$

e) $16x^2 - 25y^2$

f) $3x^3 - 12x$

7. Laws of Indices

Simplify the following:

a) $4x^5 \times 3x^3$

b) $\dfrac{15y^8}{5y^2}$

c) $(2a^4)^3 \times 3a^2$

d) $\dfrac{(3x)^4}{9x^2}$

e) $2x^3y^2 \times 4x^2y^5$

8. Indices - Fractional and Negative Powers

Simplify the following:

a) $\dfrac{x^5}{x^{-3}}$

b) $x^{\frac{1}{4}} \times x^{\frac{1}{2}}$

c) $(x^{\frac{2}{3}})^6$

d) $\sqrt{81x^8}$

e) $\sqrt[3]{64x^9}$

f) $\dfrac{x^3}{\sqrt[3]{x}}$

9. Writing in the form ax^b

Write the following in the form ax^b where a and b are constants:

a) $\dfrac{2}{x^4}$

b) $\dfrac{1}{5x^2}$

c) $\dfrac{3}{\sqrt[3]{x}}$

d) $\dfrac{2x^2}{5x^6}$

e) $\dfrac{5}{2x\sqrt{x}}$

10. Indices and Fractions

Simplify the following leaving all terms in the form ax^b where a and b are constants:

a) $\dfrac{4x^2 - 2x}{8x^4}$

b) $\dfrac{5 + 2x^4}{x^2}$

c) $\dfrac{x^2 - 3x}{\sqrt{x}}$

d) $\dfrac{(3x-1)^2}{x^3}$

11. Simplifying Surds

Simplify each of the following expressions:

a) $\sqrt{20}$

b) $\dfrac{\sqrt{72}}{3}$

c) $\sqrt{48} - 2\sqrt{27} + 4\sqrt{12}$

d) $(2 + \sqrt{3})(4 - \sqrt{3})$

e) $(2\sqrt{5} - \sqrt{2})(\sqrt{5} + 3\sqrt{2})$

12. Rationalising the denominator

Rationalise and simplify the following fractions:

a) $\dfrac{5}{\sqrt{2}}$

b) $\dfrac{6\sqrt{5}}{\sqrt{3}}$

c) $\dfrac{2}{4 - \sqrt{3}}$

d) $\dfrac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

e) $\dfrac{5}{(2 + \sqrt{5})^2}$

13. Finding the gradient

a) Find the gradient of the line joining $A(-3, 2)$ and $B(5, 6)$.

b) The line joining $C(1, -4)$ and $D(3, a)$ has a gradient of 2. Find a.

c) The line joining $E(-4, 7)$ and $F(b, -1)$ has a gradient of $-\dfrac{4}{3}$. Find b.

14. Equation of a Line

a) Write down the gradient and y-intercept of $y = -4x + 7$.

b) Write down the gradient and y-intercept of $4x - 2y + 5 = 0$.

c) Write $y = -3x + 2$ in the form $ax + by + c = 0$.

d) Write $y = \dfrac{2}{5}x - \dfrac{1}{2}$ in the form $ax + by + c = 0$.

15. Roots of Lines

Find the coordinates of the point where these lines meet the x-axis:

a) $y = 4x - 8$

b) $3x - 5y + 15 = 0$

c) $2y = 7x + 14$

16. Finding the equation of a line

a) Find the equation of the line with gradient 4 passing through $(-2, 5)$.

b) Find the equation of the line passing through $(3, 2)$ and $(5, -6)$.

c) Find, in the form $ax + by + c = 0$, the equation of the line passing through $(2, -\frac{1}{3})$ and $(4, -1)$.

17. Sketching Lines

Sketch the following lines, labelling the points of intersection with the coordinate axes:

a) $y = 3x - 6$ **b)** $3x - 2y + 12 = 0$ **c)** $4x + 5y - 20 = 0$

18. Parallel and perpendicular lines

a) Find the equation of the line parallel to $y = 3x - 4$ passing through $(2, 5)$.

b) Find the equation of the line perpendicular to $y = 3x - 4$ passing through $(6, -1)$, in the form $ax + by + c = 0$.

c) Find the equation of the line parallel to $2x + 3y - 5 = 0$ passing through $(-3, 4)$.

19. Solving Quadratic Equations via. Factorising

Solve the following equations:

a) $x^2 + 8x + 15 = 0$ **c)** $x^2 - 49 = 0$ **e)** $3x^2 + 10x - 8 = 0$

b) $x^2 - 4x - 21 = 0$ **d)** $25 - x^2 = 0$ **f)** $4x^2 - 12x + 9 = 0$

20. Sketching Quadratics

Sketch the following graphs, labelling any intersections with the axes:

a) $y = x^2 - 7x$ **b)** $y = -x^2 - 2x + 15$ **c)** $y = 2x^2 - 5x - 3$

21. The Quadratic Formula

a) Solve $3x^2 + 5x + 1 = 0$.

b) Solve $2x^2 - 3x - 4 = 0$ leaving your answer in surd form.

c) Solve $5x^2 - 10x = 3$ leaving your answer in simplified surd form.

22. Completing the square

Write the following expressions in the form $p(x + q)^2 + r$:

a) $x^2 + 6x$ **c)** $x^2 + 4x + 7$ **e)** $3x^2 + 12x + 2$

b) $x^2 - 9x$ **d)** $x^2 - 10x - 3$ **f)** $-x^2 - 6x + 5$

23. Finding turning points

Find the coordinates of the turning point of the following graphs and state the line of symmetry:

a) $y = x^2 + 6x - 2$

b) $y = x^2 - 8x + 1$

c) $y = 2x^2 + 12x - 5$

24. Solving quadratics by completing the square

For the following equations, solve by completing the square (leaving answers in surd form) and sketch the graph labelling intersections and the vertex:

a) $x^2 + 6x + 4 = 0$

b) $x^2 - 4x - 1 = 0$

c) $-2x^2 + 12x - 5 = 0$

25. Solving linear simultaneous equations

Solve the following pairs of simultaneous equations:

a) $3x + 2y = 13$ and $y = 2x - 4$

b) $5x - 2y = 11$ and $x = y + 4$

c) $3x + 4y = 2$ and $2x - y = 5$

26. Simultaneous equations with one linear equation and one non-linear equation

Solve the following pairs of simultaneous equations:

a) $y = x^2$ and $y = 2x + 3$

b) $x + y = 2$ and $x^2 + y^2 = 10$

c) $2x - y = 5$ and $x^2 - 2xy + y^2 = 4$

27. Solving linear inequalities

Find the set of values for which:

a) $3x - 4 < 11$

b) $5 - 2x \geq 9$

c) $4 - 3(x + 2) \leq 2(1 - 4x)$

d) $\dfrac{2x - 1}{3} > x + 4$

QUESTIONS TO SUBMIT (FURTHER MATHS ONLY)

Please write your full solutions on A4 paper to hand in at induction. This work must be **separate** from the A level Maths work.

Remember that A level Further Mathematics is an additional A level to Maths. You must score highly on the Maths Skills Assessment as well as complete the following summer work to be suitable for this course. Although the following questions are challenging, they are based on GCSE skills and knowledge; you should know everything you need to know already, but they will require some **problem-solving** skills. What we are looking for is how well you can communicate mathematically. A list of correct answers will score you some marks, but **well explained**, **coherent** and **concise** solutions will score you many more marks.

1 Simplify these expressions as far as possible.

a $\dfrac{x^2-2x-3}{x^2+2x+1}$ **(3 marks)**

b $\dfrac{x^2-25}{x^2+6x+8} \div \dfrac{x^2-2x-15}{x^2-16}$ **(4 marks)**

2 The line *l* is a tangent to the circle $x^2+y^2=20$ at the point *P*(2, 4).

The tangent intersects the *y*-axis at point *A*. Find the area of the triangle *OPA*. **(5 marks)**

3 Expand and simplify $\left(\sqrt{p}+2\sqrt{q}\right)\left(2\sqrt{p}-\sqrt{q}\right)$ **(3 marks)**

4 **a** Write $3x^2-12x+7$ in the form $a(x+b)^2+c$ **(3 marks)**

b Hence, or otherwise, write down the coordinates of the turning point of the graph of $y=3x^2-12x+7$ **(1 mark)**

5 Prove algebraically that the product of three consecutive **odd** numbers is always an odd number. **(4 marks)**

6 The functions g and f are defined as $\mathrm{g}(x)=\dfrac{2x}{4-x}$ and $\mathrm{f}(x)=3x-1$

Given that $x \neq 4$, find the value(s) of *x* such that $\mathrm{g}(x)=\mathrm{f}(x)$, giving your answer(s) to 2 decimal places. **(6 marks)**

7 The line l_1 has equation $y = -\frac{1}{2}x + 3$ and intersects the *x*- and *y*-axes at the points *A* and *B* respectively.

a Find the exact length of the line segment *AB*. **(3 marks)**

b Find the equation of the line l_2 perpendicular to l_1 which passes through the point *P*(−1, −2). **(2 marks)**

The line l_2 intersects l_1 at the point *C*.

c Find the midpoint of the line segment *AC*. **(4 marks)**

8 A triangle *ABC* has side lengths *AB* = 10 cm, *BC* = 15 cm and *AC* = 8 cm.

a Find the size of the largest angle, giving your anwer to 2 decimal places. **(3 marks)**

b Find the area of the triangle, giving your anwer to 2 decimal places. **(2 marks)**

9 **a** Sketch the graph of $y = \cos x$ for $-180 \leqslant x \leqslant 360°$, showing the points where the graph cuts the axes. **(2 marks)**

b Hence find the exact values of *x* in the interval $-180 \leqslant x \leqslant 360°$ for which $\cos x = -\frac{\sqrt{3}}{2}$ **(3 marks)**

THIS WORK IS DUE FOR:

Your Induction Lesson. Please ensure you bring it with you!!

If there are any questions about this work, you can email:

Martin Hickman: m.hickman@barnsley.ac.uk